FITTING OF MITSCHERLICH'S CURVE TO UNEQUALLY SPACED LEVELS

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1. Introduction

Stevens (1951) has given a method for fitting the asymptotic regression law $y=\alpha+\beta\rho^{x}$ both for equally and unequally spaced values of x. The necessary tables for computing the estimates as well as variances of the estimates were given by him for the 5, 6 and 7 equally spaced levels of x. Additional tables for 11, 20 and 40 levels were computed by Linhart (1960), while for 3 to 12 equally spaced levels Lipton (1962) prepared the necessary tables. This procedure can be used in case of fitting Mitscherlich's curve by suitably transforming the various parameters involved.

The general method of estimating the parameters in the Mitscherlich's response equation has been described by Pimental Gomes (1953) in case of equally spaced levels and values of the polynomials involved in the solution have been tabulated for 4 and 5 equally spaced levels.

When the results of several experiments are summarised to study the response curves, one often encounters the situation where the levels tried are not equally spaced. Inkson (1964) has indicated the solution for suitations involving sets of coded fertilizer levels :— (*i*) 0, 1, 2 ; (*ii*) 0, 1, 3 ; (*iii*) 0, 1, 4 ; (*iv*) 0, 1, 2, 3 ; (*v*) 0, 1, 2, 4 ; and (*vi*) 0, 1, 4, 8 without providing the tables. The procedure of estimating the parameters of the Mitscherlich's response equation along-with tables, wherever necessary, of values of polynomials involved in the solution for situations involving the coded doses—(*i*) 0, 1, 3 ; (*ii*) 0, 2, 3 ; (*iii*) 0, 1, 2, 4 ; (*iv*) 0, 1, 3, 4 ; and (*v*) 0, 2, 3, 4 are presented in this paper.

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2. Estimation of Parameters

The method indicated by Pimental Gomes (1953) will be followed for the estimation of parameters. Let y_m denote the yield obtained with fertilizer at x_m ($=mq$). The estimates of the parameters are obtained by minimising

$$W=\Sigma[y-A\{1-10^{-c(x+B)}\}]^2$$

which leads to the following set of equations :—

$$\begin{aligned}
&\sum_m y_m && -S.A && +A.\ 10^{-Bc}\sum_m Z^m=0\\
&\sum_m m.y_m Z^m && -A\sum_m m.Z^m && +A.\ 10^{-Bc}\sum_m mZ^{2m}=0\\
&\sum_m y_m.Z^m && -A\sum_m Z^m && +A.\ 10^{-Bc}\sum_m Z^{2m}=0
\end{aligned} \quad \text{.. (1)}$$

where $Z=10^{-cq}$ and S is the number of levels of fertiliser.

Set (1) leads to the equation

$$\begin{vmatrix}
\sum_m y_m & S & \sum_m Z^m\\
\sum_m m.y_m.Z^m & \sum_m m.Z^m & \sum_m m.Z^{2m}\\
\sum_m y_m.Z^m & \sum_m Z^m & \sum_m Z^{2m}
\end{vmatrix}=0 \quad \text{...(2)}$$

This equation (2) depends only upon 'Z' and hence only the parameter 'C'. Since both c and q are positive, equation (2) may be solved for 'Z' lying between zero and 1. If the equation has no root between zero and 1, then the fitting of the curve cannot be carried out.

Equation (2) can be reduced to the following form keeping in view that the accepted root lies between zero and 1 :

$$R(Z)=\Sigma_m\ y_m\ .\ J_{sm}(Z)=0 \quad \text{.. (3)}$$

where $J_{sm}(Z)$'s are polynomials in Z with $\sum_m J_{sm}(Z)=0$,

When a solution of Z (say z) is obtained between zero and 1, the parameter 'c' can be estimated by $\hat{c}=(1/q)\log(1/z)$. The estimates $\hat{A}$ and $\hat{B}$ can then be obtained from set (1). The variances of these estimates can be computed following the procedure outlined by Stevens (1951).

The expression $R(Z)$ alongwith the estimates of the parameters are given below for different cases considered in the paper.

Case (i) *Coded levels 0, 1, 3* :

In this case we have

$$R(Z)=y_0(Z+Z^2)+y_1(-1-Z-Z^2)+y_3. \qquad \text{.. (4)}$$

The equation $R(Z)=0$, being a second degree equation, can easily be solved for the solution 'z' lying between zero and 1. The estimates $\hat{A}$ and $\hat{B}$ are given by :—

$$\hat{A}=\frac{y_0(-z-z^3-z^4-z^5)+y_1(1-z^4-z^5)+y_3(1+z+2z^2+z^3)}{2(1-z)(1+z+2z^2+2z^3+z^4)}$$

and $$\hat{B}=(1/\hat{c})\log\frac{1+z+z^3}{3-(1/\hat{A})(y_0+y_1+y_3)} \qquad \text{...(5)}$$

Case (ii) *Coded levels 0,* 2, *3* :

We have

$$R(Z)=y_0(Z^2)+y_2(-1-Z-Z^2)+y_3(1+Z) \qquad \text{...(6)}$$

in this case. The equation $R(Z)=0$, being a second degree equation in this case also, can easily be solved for the solution z, lying between zero and 1. The estimates $\hat{A}$ and $\hat{B}$ can be obtained as

$$\hat{A}=\frac{y_0(-z^2)(1+2z+z^2+z^3)+y_2(1+z-z^5)+y_3(1+z+z^2+z^4)}{2(1-z)(1+2z+2z^2+z^3+z^4)}$$

and $$\hat{B}=(1/\hat{c})\log\frac{1+z^2+z^3}{3-(1/\hat{A})(y_0+y_2+y_3)}. \qquad \text{...(7)}$$

Case (iii) *Coded levels 0, 1, 2, 4* :

$$\begin{aligned}R(Z)=&y_0(2Z+3Z^2+8Z^3+11Z^4+9Z^5+2Z^6)\\&+y_1(-2-4Z-6Z^2-6Z^3+4Z^4+2Z^6)\\&+y_2(1-2Z-6Z^2-9Z^3-11Z^4-9Z^5-4Z^6)\\&+y_4(1+4Z+9Z^2+7Z^3+4Z^4)\end{aligned} \qquad \text{...(8)}$$

in this case. The equation $R(Z)=0$, in this case, being of 6th degree would be difficult to solve for a solution z, lying between zero and 1 directly. To facilitate the solution, the polynomial co-efficients have been tabulated (Appendix I) for values of z lying between zero and 1 at intervals of 0·01. The estimates $\hat{A}$ and $\hat{B}$ are obtained as :

$$\hat{A}=\frac{y_0(-z-z^2-z^3-z^4-z^5-z^6-z^7)+y_1(1-z^3-z^5-z^6-z^7)+y_2(1+z+z^2-z^6-z^7)+y_4(1+z+2z^2+2z^3+2z^4+z^5)}{(1-z)\,(3+4z+6z^2+6z^3+7z^4+6z^5+3z^6)}$$

and

$$\hat{B}=(1/\hat{c})\,\log\frac{1+z+z^2+z^4}{4-(1/\hat{A})(y_0+y_1+y_2+y_4)} \qquad ...(9)$$

Case (iv) *Coded levels 0, 1, 3, 4.*

We have

$$R(Z)=y_0(2Z+6Z^2+8Z^3+5Z^4+6Z^5+2Z^6+Z^7)+y_1(-2-6Z-9Z^2-8Z^3-5Z^4-3Z^5+2Z^6+Z^7)+y_3(1+2Z-3Z^2-5Z^3-8Z^4-9Z^5-6Z^6-2Z^7)+y_4(1+2Z+6Z^2+5Z^3+8Z^4+6Z^5+2Z^6) \qquad ...(10)$$

in this case. Thus the equation $R(Z)=0$ will in this case be of 7th degree and it would be difficult to solve it for a solution z, lying between zero and 1. To facilitate the solution the polynomial coefficients have been tabulated (Appendix II) as for the case (*iii*).

The estimates $\hat{A}$ and $\hat{B}$ are given by

$$\hat{A}=\frac{y_0(-z-z^3-2z^4-2z^5-z^6-z^7)+y_1(1-z^4-2z^5-z^6-z^7)+y_3(1+z+2z^2+z^3-z^7)+y_4(1+z+2z^2+2z^3+z^4+z^6)}{(1-z)\,(3+4z+8z^2+10z^3+8z^4+4z^5+3z^6)}$$

and $$\hat{B}=(1/\hat{c})\,\log\frac{1+z+z^3+z^4}{4-(1/\hat{A})(y_0+y_1+y_3+y_4)} \qquad ...(11)$$

Case (v) *Coded levels 0, 2, 3, 4.*

In this case

$$R(Z)=y_0(4Z^2+7Z^3+9Z^4+4Z^5+Z^6)+y_2(-4-9Z-11Z^2-9Z^3-6Z^4-2Z^5+Z^6)+y_3(2-4Z^2-6Z^3-6Z^4-4Z^5-2Z^6)+y_4(2+9Z+11Z^2+8Z^3+3Z^4+2Z^5) \qquad ...(12)$$

The equation $R(Z)=0$ will therefore be one of 6th degree. To facilitate solution, the polynomial co-efficients have been tabulated (Appendix III) as in cases (*iii*) and (*iv*). The estimates $\hat{A}$ and $\hat{B}$ can be obtained as

$$\hat{A}=\frac{y_0(-z^2)(1+z+z^2)(1+z+z^3)+y_2(1+z-z^5-z^6-z^7)+y_3(1+z+z^2)(1+z^4-z^5)+y_4(1+z+z^2+z^3+z^4+z^5+z^6)}{(1-z)(3+6z+7z^2+6z^3+6z^4+4z^5+3z^6)}$$

and $$\hat{B}=(1/\hat{c})\log\frac{1+z^2+z^3+z^4}{4-(1/\hat{A})(y_0+y_2+y_3+y_4)} \qquad \ldots(13)$$

SUMMARY

The solution of Mitscherlich's response equation by the method of Pimental Gomes (1953) has been discussed for several situations involving unequally spaced levels. To facilitate the solution, the polynomial co-efficients involved in the equations have been tabulated, wherever necessary.

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APPENDIX I

Polynomial co-efficients for the situation involving coded doses 0, 1, 2, 4.

z	$J_{40}(z)$	$J_{41}(z)$	$J_{42}(z)$	$J_{44}(z)$
0.00	0.0000	−2.0000	1.0000	1. 000
0.01	0.0203	−2.0406	0.9794	1.0409
0.02	0.0413	−2.0824	0·9575	1.0837
0.03	0.0629	−2.1256	0.9343	1.1283
0.04	0.0853	−2.1700	0.9098	1.1749
0.05	0.1086	−2.2158	0.8838	1.2234
0 06	0.1327	−2.2629	0.8563	1.2740
0.07	0.1577	−2.3116	0.8272	1.3266
0.08	0.1838	−2.3616	0.7965	1.3813
0.09	0.2109	−2.4132	0.7641	1.4383
0.10	0.2392	−2.4664	0.7298	1.4974
0.11	0.2687	−2 5212	0.6937	1.5588
0.12	0.2995	−2.5776	0.6555	1.6225
0.13	0.3318	−2.6357	0.6153	1.6886
0.14	0.3655	−2.6956	0.5730	1.7571
0.15	0.4008	−2.7573	0.5283	1.8282
0.16	0.4378	−2.8208	0.4813	1.9017
0.17	0.4765	−2.8862	0.4318	1.9778
0.18	0.5172	−2.9535	0.3797	2.0566
0.19	0.5598	−3.0229	0.3249	2.1381
0.20	0.6046	−3.0943	0.2673	2.2224
0.21	0.6516	−3.1678	0.2066	2.3095
0.22	0.7010	−3.2434	0.1429	2.3995
0.23	0.7529	−3.3213	0.0759	2.4925
0.24	0.8074	−3.4014	0.0056	2.5884
0.25	0.8647	−3.4839	−0.0684	2.6875
0.26	0.9250	−3.5687	−0.1460	2.7897
0.27	0.9883	−3.6560	−0.2275	2.8951
0.28	1.0549	−3.7457	−0.3130	3.0039
0.29	1.1249	−3.8380	−0.4027	3.1159
0.30	1.1984	−3.9329	−0.4969	3.2314

0.31	1.2758	−4.0305	−0.5956	3.3504
0.32	1.3570	−4.1308	−0.6991	3.4729
0.33	1.4425	−4.2339	−0.8077	3.5991
0.34	1.5322	−4.3398	−0.9214	3.7290
0.35	1.6265	−4.4486	−1.0406	3.8626
0.36	1.7256	−4.5604	−1.1654	4.0002
0.37	1.8296	−4.6752	−1.2961	4.1416
0.38	1.9389	−4.7930	−1.4330	4.2871
0 39	2.0536	−4.9140	−1.5762	4.4367
0.40	2.1740	−5.0382	−1.7261	4.5904
0.41	2.3003	−5.1657	−1.8830	4.7484
0.42	2.4328	−5.2964	−2.0471	4.9107
0.43	2.5718	−5.4306	−2.2186	5.0774
0.44	2.7175	−5.5681	−2.3980	5.2486
0.45	2.8703	−5.7092	−2.5855	5.4244
0.46	3.0303	−5.8538	−2.7814	5.6049
0.47	3.1980	−6.0020	−2.9861	5.7900
0.48	3.3736	−6.1538	−3.1999	5.9801
0.49	3.5575	−6.3094	−3.4232	6.1750
0.50	3.7500	−6.4688	−3.6562	6.3750
0.51	3.9514	−6.6319	−3.8995	6.5801
0.52	4.1621	−6.7990	−4.1534	6.7903
0.53	4.3824	−6.9700	−4.4183	7.0059
0.54	4.6127	−7.1449	−4.6945	7.2268
0.55	4.8534	−7.3239	−4 9826	7.4532
0.56	5.1049	−7.5070	−5.2830	7.6851
0.57	5.3675	−7.6942	−5.5960	7.9227
0.58	5.6418	−7.8856	−5.9222	8.1660
0.59	5.9280	−8.0812	−6.2621	8.4152
0.60	6.2268	−8.2811	−6.6161	8.6704
0.61	6.5384	−8.4853	−6.9847	8.9316
0.62	6.8633	−8.6938	−7.3685	9.1989
0.63	7.2021	−8.9068	−7.7679	9.4725
0.64	7.5553	−9.1241	−8.1836	9.7525
0.65	7.9232	−9.3459	−8.6161	10.0389
0.66	8.3064	−9.5723	−9.0660	10.3319
0.67	8.7055	−9.8031	−9.5338	10.6315
0.68	9.1209	−10.0385	−10.0202	10.9379

0.69	9.5532	−10.2785	−10.5259	11.2511
0.70	10.0030	−10.5231	−11.0513	11.5714
0.71	10.4709	−10.7723	−11.5973	11.8987
0.72	10.9574	−11.0262	−12.1644	12.2333
0.73	11.4631	−11.2848	−12.7535	12.5751
0.74	11.9886	−11.5480	−13.3651	12.9244
0.75	12.5347	−11.8159	−14.0000	13.2812
0.76	13.1018	−12.0885	−14.6590	13.6457
0.77	13.6907	−12.3659	−15.3428	14.0180
0.78	14.3021	−12.6479	−16.0523	14 3981
0.79	14.9366	−12.9347	−16.7882	14.7862
0.80	15.5950	−13.2261	−17.5513	15.1824
0.81	16.2779	−13.5223	−18.3425	15.5869
0.82	16.9862	−13.8231	−19.1627	15.9997
0.83	17.7204	−14.1286	−20.0128	16.4209
0.84	18.4815	−14.4387	−20.8936	16.8508
0.85	19.2702	−14.7535	−21.8061	17.2894
0.86	20.0873	−15.0728	−22.7513	17.7368
0.87	20.9336	−15.3968	−23.7301	18.1932
0.88	21.8100	−15.7252	−24.7435	18.6587
0.89	22.7173	−16.0581	−25.7925	19.1334
0.90	23.6564	−16.3955	−26.8783	19 6174
0.91	24.6281	−16.7373	−28.0018	20.1109
0.92	25.6335	−17.0834	−29.1641	20.6140
0.93	26.6733	−17.4338	−30.3663	21.1268
0.94	27.7486	−17.7884	−31.6097	21.6495
0.95	28.8603	−18.1471	−32.8953	22.1822
0.96	30.0094	−18.5099	−34.2244	22.7249
0.97	31.1969	−18.8767	−35.5982	23.2780
0.98	32.4238	−19.2473	−37.0178	23.8414
0.99	33.6911	−19.6218	−38.4847	24.4154
1.00	35.0000	−20.0000	−40.0000	25.0000

APPENDIX II

Polynomial co-efficients for the situation involving coded doses 0, 1, 3, 4.

z	$J_{40}(z)$	$J_{41}(z)$	$J_{43}(z)$	$J_{44}(z)$
0.00	0.0000	—2.0000	1.0000	1.0000
0.01	0.0206	—2.0609	1.0197	1.0206
0.02	0.0425	—2.1237	1.0388	1.0424
0.03	0.0656	—2.1883	1.0572	1.0655
0.04	0.0901	—2.2549	1.0749	1.0899
0.05	0.1160	—2.3235	1.0918	1.1157
0.06	0.1434	—2.3942	1.1080	1.1428
0.07	0.1723	—2.4670	1.1234	1.1713
0.08	0.2027	—2.5419	1.1379	1.2013
0.09	0.2348	—2.6191	1.1515	1.2328
0.10	0.2686	—2.6985	1.1641	1.2659
0.11	0.3041	—2.7803	1.1757	1.3005
0.12	0.3414	—2.8645	1.1863	1.3369
0.13	0.3806	—2.9512	1.1957	1.3749
0.14	0.4218	—3.0404	1.2039	1.4147
0.15	0.4650	—3.1322	1.2108	1.4564
0.16	0.5103	—3.2267	1.2164	1.5000
0.17	0.5578	—3.3240	1.2206	1.5455
0.18	0.6075	—3.4240	1.2233	1.5932
0.19	0.6596	—3.5269	1.2245	1.6429
0.20	0.7141	—3.6328	1.2239	1.6948
0.21	0.7711	—3.7417	1.2216	1.7491
0.22	0.8306	—3.8538	1.2175	1.8057
0.23	0.8929	—3.9690	1.2113	1.8648
0.24	0.9580	—4.0875	1.2031	1.9264
0.25	1.0259	—4.2094	1.1927	1.9907
0.26	1.0969	—4.3347	1.1801	2.0572
0.27	1.1709	—4.4636	1.1649	2.1277
0.28	1.2482	—4.5960	1.1472	2.2006
0.29	1.3287	—4.7322	1.1268	2.2766
0.30	1.4128	—4.8721	1.1035	2.3558

0.31	1.5003	−5.0159	1.0772	2.4384
0.32	1.5916	−5.1637	1.0477	2.5244
0.33	1.6867	−5.3156	1.0149	2.6140
0.34	1.7857	−5.4717	0.9786	2.7074
0.35	1.8889	−5.6320	0.9385	2.8046
0.36	1.9962	−5.7966	0.8945	2.9059
0.37	2.1080	−5.9658	0.8464	3 0113
0.38	2.2243	−6.1394	0.7940	3.1211
0.39	2.3454	−6.3178	0.7370	3.2354
0.40	2.4713	−6.5009	0 6752	3.3544
0.41	2.6022	−6.6889	0.6084	3.4783
0.42	2.7384	−6.8818	0.5363	3.6072
0.43	2.8800	−7.0798	0.4586	3 7413
0.44	3.0271	−7.2830	0.3751	3.8808
0.45	3.1801	−7.4915	0.2855	4.0260
0.46	3.3390	−7.7054	0.1894	4.1770
0.47	3.5042	−7.9248	0.0866	4.3341
0.48	3.6758	−8 1499	−0.0233	4.4974
0.49	3.8540	−8.3806	−0.1406	4.6672
0.50	4.0391	−8.6172	−0.2656	4.8438
0.51	4.2312	−8.8597	−0.3988	5.0273
0.52	4.4308	−9.1083	−0.5405	5.2180
0.53	4.6379	−9.3630	−0 6912	5.4163
0.54	4.8529	−9.6240	−0.8512	5.6223
0.55	5.0761	−9.8914	−1.0209	5.8363
0.56	5.3076	−10.1653	−1.2009	6.0586
0.57	5.5479	−10.4458	−1.3915	6.2895
0.58	5.7972	−10.7330	−1.5934	6.5[illegible]92
0.59	6.0557	−11.0270	−1.8069	6.7782
0.60	6.3239	−11.3280	−2.0326	7.0367
0.61	6.6020	−11.6360	−2.2710	7.3050
0.62	6.8903	−11.9511	−2.5227	7.5834
0.63	7.1893	−12.2734	−2.7883	7.8724
0.64	7.4993	−12.6031	−3.0683	8.1722
0.65	7.8206	−12.9403	−3.3635	8.4832
0.66	8.1536	−13.2849	−3.6744	8.8058
0.67	8.4987	−13.6373	−4.0017	9.1403
0.68	8.8562	−13.9973	−4.3461	9.4872

0.69	9.2267	−14.3652	−4.7083	9.8468
0.70	9.6106	−14.7411	−5.0890	10.2195
0.71	10.0082	−15.1249	−5.4891	10.6058
0.72	10.4200	−15.5168	−5.9093	11.0061
0.73	10.8464	−15.9169	−6.3504	11.4209
0.74	11.2881	−16.3253	−6.8132	11.8505
0.75	11.7453	−16.7420	−7.2987	12.2954
0.76	12.2187	−17.1671	−7.8077	12.7562
0.77	12.7087	−17.6007	−8.3412	13.2332
0.78	13.2159	−18.0429	−8.9001	13.7271
0.79	13.7409	−18.4936	−9.4855	14 2382
0.80	14.2841	−18.9530	−10.0982	14.7672
0.81	14.8462	−19.4212	−10.7395	15.3145
0.82	15.4277	−19.8981	−11.4103	15.8807
0.83	16.0293	−20.3838	−12.1118	16.4663
0.84	16.6515	−20.8783	−12.8452	17.0720
0.85	17.2951	−21.3818	−13.6116	17.6982
0.86	17.9607	−21.8941	−14.4122	18.3456
0.87	18.6489	−22.4154	−15.2485	19.0149
0.88	19.3605	−22.9456	−16.1215	19.7065
0.89	20.0962	−23.4847	−17.0327	20.4212
0.90	20.8566	−24.0328	−17.9834	21.1596
0.91	21.6426	−24.5898	−18.9752	21.9224
0.92	22.4549	−25.1558	−20.0093	22.7102
0.93	23.2943	−25.7306	−21.0874	23.5237
0.94	24.1617	−26.3143	−22.2110	24.3637
0.95	25.0577	−26.9069	−23.3817	25.2308
0.96	25.9834	−27.5082	−24.6010	26.1258
0.97	26.9396	−28.1182	−25.8708	27.0495
0 98	27.9271	−28.7369	−27.1927	28.0025
0.99	28.9469	−29.3642	−28.5685	28.9858
1.00	30.0000	−30.0000	−30.0000	30.0000

APPENDIX III

Polynomial co-efficients for the situation involving coded doses 0, 2, 3, 4.

z	$J_{40}(z)$	$J_{42}(z)$	$J_{43}(z)$	$J_{44}(z)$
0.00	0.0000	−4.0000	2.0000	2.0000
0.01	0.0004	−4.0911	1.9996	2.0911
0.02	0.0017	−4.1845	1.9984	2.1845
0.03	0.0038	−4.2801	1 9962	2.2801
0.04	0.0069	−4.3782	1.9932	2.3781
0.05	0.0109	−4.4787	1.9892	2.4785
0.06	0.0160	−4.5816	1.9842	2.5814
0.07	0.0222	−4.6871	1.9782	2.6867
0.08	0.0296	−4.7953	1.9711	2.7946
0.09	0.0381	−4.9061	1.9628	2.9051
0.10	0.0479	−5.0196	1.9534	3.0183
0.11	0.0591	−5.1360	1.9427	3 1342
0.12	0.0717	−5.2552	1.9307	3.2529
0.13	0.0857	−5.3775	1.9173	3.3744
0.14	0.1013	−5.5027	1.9026	3.4988
0.15	0.1185	−5.6311	1.8864	3.6262
0.16	0.1374	−5.7626	1.8686	3.7565
0.17	0.1581	−5.8974	1.8493	3.8900
0.18	0.1807	−6.0355	1.8283	4.0266
0.19	0.2052	−6.1771	1.8055	4.1664
0.20	0.2317	−6.3222	1.7810	4.3094
0.21	0.2604	−6.4708	1.7546	4.4558
0.22	0.2914	−6.6232	1.7262	4.6056
0.23	0.3247	−6.7793	1.6958	4.7589
0.24	0.3604	−6.9393	1.6632	4.9157
0.25	0.3987	−7.1033	1.6284	5.0762
0.26	0.4396	−7.2713	1.5914	5.2403
0.27	0.4833	−7.4434	1.5519	5.4082
0.28	0.5299	−7.6198	1.5100	5.5799
0.29	0.5796	−7.8005	1.4654	5.7555
0.30	0.6323	−7.9857	1.4182	5.9352

0.31	0.6884	−8.1755	1.3682	6.1189
0.32	0.7478	−8.3699	1.3153	6.3067
0.33	0.8108	−8.5690	1.2594	6.4988
0.34	0 8775	−8.7731	1.2004	6.6952
0.35	0.9480	−8.9821	1.1380	6.8960
0.36	1.0225	−9.1962	1.0723	7.1013
0.37	1.1011	−9.4155	1.0032	7.3112
0 38	1.1841	−9.6402	0.9303	7.5258
0.39	1.2715	−9.8703	0.8538	7.7451
0.40	1.3635	−10.1060	0.7732	7.9693
0.41	1.4603	−10.3474	0.6887	8.1984
0.42	1.5620	−10.5945	0.5999	8.4326
0.43	1.6690	−10.8477	0.5068	8.6719
0.44	1.7812	−11.1069	0.4091	8.9165
0.45	1.8990	−11.3723	0.3068	9.1664
0.46	2.0226	−11.6440	0.1996	9.4218
0.47	2.1520	−11.9222	0.0874	9.6827
0.48	2.2877	−12.2070	−0.0300	9.9493
0.49	2.4296	−12.4985	−0.1529	10.2217
0.50	2.5781	−12.7969	−0.2812	10.5000
0.51	2.7334	−13.1012	−0.4154	10.7843
0.52	2.8958	−13.4148	−0.5556	11.0747
0.53	3.0653	−13.7347	−0.7021	11.3713
0.54	3.2424	−14.0620	−0.8546	11.6742
0.55	3.4272	−14.3969	−1.0140	11.9837
0.56	3.6200	−14.7395	−1.1801	12.2997
0.57	3.8210	−15.0900	−1.3534	12.6225
0.58	4.0305	−15.4486	−1.5339	12.9521
0.59	4.2488	−15.8154	−1.7220	13.2886
0.60	4.4761	−16.1905	−1.9180	13.6323
0.61	4.7128	−16.5741	−2.1219	13.9832
0.62	4.9590	−16.9664	−2.3342	14.3415
0.63	5.2152	−17.3675	−2.5551	14.7074
0.64	5.4816	−17.7776	−2.7848	15.0808
0.65	5.7585	−18.1968	−3.0237	15.4621
0.66	6.0462	−18.6254	−3.2721	15.8513
0.67	6.3451	−19.0634	−3.5302	16.2486
0.68	6.6554	−19.5111	−3.7984	16.6541

0.69	6.9775	−19.9686	−4.0769	17.0680
0.70	7.3118	−20.4361	−4.3662	17.4904
0.71	7.6586	−20.9137	−4.6665	17.9216
0.72	8.0183	−21.4017	−4.9781	18.3616
0.73	8.3911	−21.9002	−5.3015	18.8106
0.74	8.7776	−22.4094	−5.6370	19.2688
0.75	9.1780	−22.9294	−5.9849	19.7363
0.76	9.5927	−23.4605	−6.3456	20.2134
0.77	10.0222	−24.0028	−6.7195	20.7001
0.78	10.4669	−24.5565	−7.1071	21.1967
0.79	10.9271	−25.1218	−7.5086	21.7033
0.80	11.4033	−25.6988	−7.9246	22.2202
0.81	11.8958	−26.2878	−8.3554	22.7474
0.82	12.4052	−26.8889	−8.8015	23.2852
0.83	12.9319	−27.5024	−9.2633	23.8338
0.84	13.4763	−28.1283	−9.7413	24.3933
0.85	14.0389	−28.7669	−10.2359	24.9639
0.86	14.6201	−29.4184	−10.7476	25.5459
0.87	15.2205	−30.0830	−11.2769	26.1395
0.88	15.8405	−30.7609	−11 8243	26.7447
0.89	16.4806	−31.4522	−12.3903	27.3619
0.90	17.1413	−32.1571	−12.9754	27.9913
0.91	17 8231	−32.8759	−13.5802	28 6330
0.92	18.5266	−33.6088	−14.2051	29.2872
0.93	19.2533	−34.3558	−14.8508	29.9543
0.94	20.0007	−35.1173	−15.5178	30.6343
0.95	20.7724	−35.8934	−16.2066	31.3276
0.96	21.5679	−36.6843	−16.9179	32.0343
0.97	22.3879	−37.4902	−17.6523	32.7546
0.98	23.2328	−38.3113	−18.4103	33.4889
0.99	24.1033	−39.1479	−19.1927	34.2373
1.00	25.0000	−40.0000	−20.0000	35.0000